4-1 Congruent Figures

Objective To recognize congruent figures and their corresponding parts
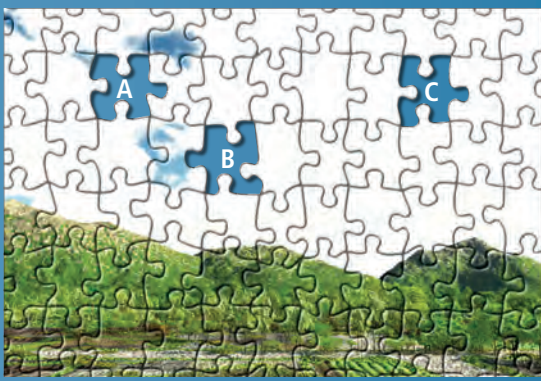

SOLVE IT!

Getting Ready!

You are working on a puzzle. You've almost finished, except for a few pieces of the sky. Place the remaining pieces in the puzzle. How did you figure out where to place the pieces?

Having trouble? How can tracing pieces 1, 2, and 3 help?

MATHEMATICAL PRACTICES

Lesson Vocabulary
• congruent polygons
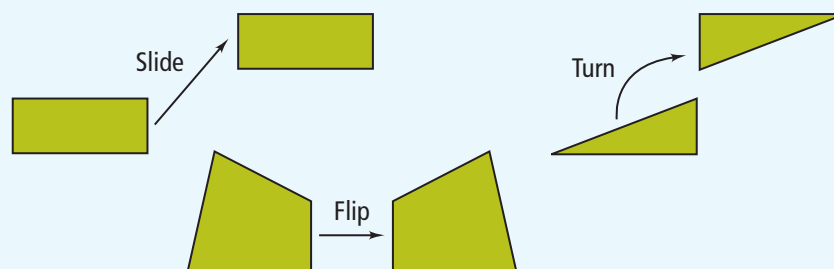
Congruent figures have the same size and shape. When two figures are congruent, you can slide, flip, or turn one so that it fits exactly on the other one, as shown below. In this lesson, you will learn how to determine if geometric figures are congruent.

Slide

Flip

Turn

Essential Understanding You can determine whether two figures are congruent by comparing their corresponding parts.
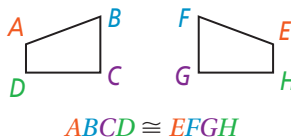
take note

Key Concept Congruent Figures

Definition

Congruent polygons have congruent corresponding parts—their matching sides and angles. When you name congruent polygons, you must list corresponding vertices in the same order.

Example

$ABCD \cong EFGH$

$\overline{AB} \cong \overline{EF}$ $\overline{BC} \cong \overline{FG}$
$\overline{CD} \cong \overline{GH}$ $\overline{DA} \cong \overline{HE}$

$\angle A \cong \angle E$ $\angle B \cong \angle F$
$\angle C \cong \angle G$ $\angle D \cong \angle H$

Plan

How do you know which sides and angles correspond?
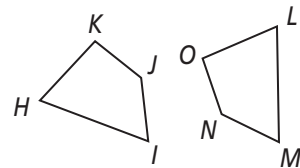The congruence statement $HIJK \cong LMNO$ tells you which parts correspond.

Problem 1   Finding Congruent Parts

If $HIJK \cong LMNO$, what are the congruent corresponding parts?

Sides: $\overline{HI} \cong \overline{LM}$ $\overline{IJ} \cong \overline{MN}$ $\overline{JK} \cong \overline{NO}$ $\overline{KH} \cong \overline{OL}$

Angles: $\angle H \cong \angle L$ $\angle I \cong \angle M$ $\angle J \cong \angle N$ $\angle K \cong \angle O$

Got It? **1.** If $\triangle WYS \cong \triangle MKV$, what are the congruent corresponding parts?

Plan

You know two angle measures in $\triangle ABC$. How can they help?
In the congruent triangles, $\angle D$ corresponds to $\angle A$, so you know that $\angle D \cong \angle A$. You can find $m\angle D$ by first finding $m\angle A$.

Problem 2   Using Congruent Parts

Multiple Choice The wings of an SR-71 Blackbird aircraft suggest congruent triangles. What is $m\angle D$?

(A) 30   (B) 75   (C) 105   (D) 150

Think

Use the Triangle Angle-Sum Theorem to write an equation involving $m\angle A$.
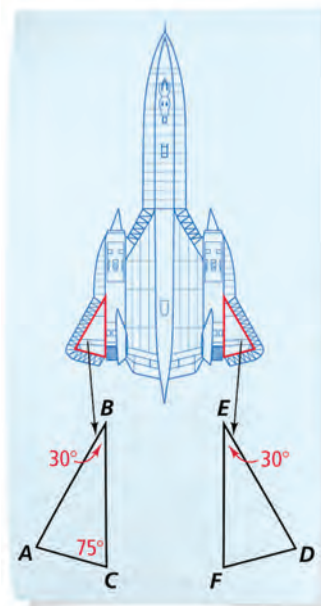
Solve for $m\angle A$.

$\angle A$ and $\angle D$ are corresponding parts of congruent triangles, so $\angle A \cong \angle D$.

Write

$m\angle A + 30 + 75 = 180$

$m\angle A + 105 = 180$
$m\angle A = 75$

$m\angle A = m\angle D = 75$

The correct answer is B.

Got It? **2.** Suppose that $\triangle WYS \cong \triangle MKV$. If $m\angle W = 62$ and $m\angle Y = 35$, what is $m\angle V$? Explain.
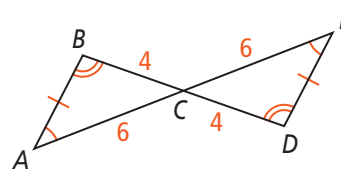
Problem 3   **Finding Congruent Triangles**

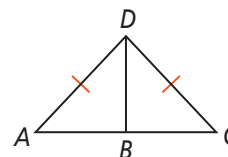**Are the triangles congruent? Justify your answer.**

$\overline{AB} \cong \overline{ED}$ | Given
$\overline{BC} \cong \overline{DC}$ | $BC = 4 = DC$
$\overline{AC} \cong \overline{EC}$ | $AC = 6 = EC$
$\angle A \cong \angle E, \angle B \cong \angle D$ | Given
$\angle BCA \cong \angle DCE$ | Vertical angles are congruent.

$\triangle ABC \cong \triangle EDC$ by the definition of congruent triangles.

Got It?   **3.** Is $\triangle ABD \cong \triangle CBD$? Justify your answer.

Recall the Triangle Angle-Sum Theorem: The sum of the measures of the angles in a triangle is 180. The next theorem follows from the Triangle Angle-Sum Theorem.

take note

Theorem 4-1   Third Angles Theorem

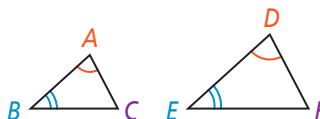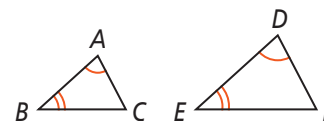| **Theorem**	**If . . .**	**Then . . .**
If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.	$\angle A \cong \angle D$ and $\angle B \cong \angle E$	$\angle C \cong \angle F$

Proof **Proof of Theorem 4-1: Third Angles Theorem**

Given: $\angle A \cong \angle D, \angle B \cong \angle E$

Prove: $\angle C \cong \angle F$

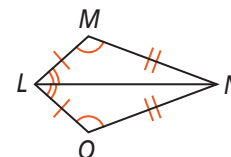| **Statements**	**Reasons**
1) $\angle A \cong \angle D, \angle B \cong \angle E$	**1)** Given
2) $m\angle A = m\angle D, m\angle B = m\angle E$	**2)** Def. of $\cong$ $\angle$s
3) $m\angle A + m\angle B + m\angle C = 180,$ $m\angle D + m\angle E + m\angle F = 180$	**3)** $\triangle$ Angle-Sum Thm.
4) $m\angle A + m\angle B + m\angle C = m\angle D + m\angle E + m\angle F$	**4)** Subst. Prop.
5) $m\angle D + m\angle E + m\angle C = m\angle D + m\angle E + m\angle F$	**5)** Subst. Prop.
6) $m\angle C = m\angle F$	**6)** Subtraction Prop. of $=$
7) $\angle C \cong \angle F$	**7)** Def. of $\cong$ $\angle$s

Proof **Problem 4** **Proving Triangles Congruent**

Given: $\overline{LM} \cong \overline{LO}$, $\overline{MN} \cong \overline{ON}$, $\angle M \cong \angle O$, $\angle MLN \cong \angle OLN$
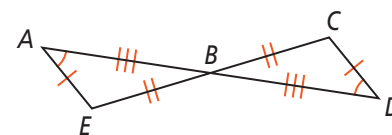
Prove: $\triangle LMN \cong \triangle LON$

Statements	Reasons
1) $\overline{LM} \cong \overline{LO}$, $\overline{MN} \cong \overline{ON}$	1) Given
2) $\overline{LN} \cong \overline{LN}$	2) Reflexive Property of $\cong$
3) $\angle M \cong \angle O$, $\angle MLN \cong \angle OLN$	3) Given
4) $\angle MNL \cong \angle ONL$	4) Third Angles Theorem
5) $\triangle LMN \cong \triangle LON$	5) Definition of $\cong$ triangles

Got It? **4. Given:** $\angle A \cong \angle D$, $\overline{AE} \cong \overline{DC}$, $\overline{EB} \cong \overline{CB}$, $\overline{BA} \cong \overline{BD}$

Prove: $\triangle AEB \cong \triangle DCB$

Lesson Check

Do you know HOW?

Complete the following statements.

1. **Given:** $\triangle QXR \cong \triangle NYC$
   a. $\overline{QX} \cong$?
   b. $\angle Y \cong$?

2. **Given:** $\triangle BAT \cong \triangle FOR$
   a. $\overline{TA} \cong$?
   b. $\angle R \cong$?

3. **Given:** $BAND \cong LUCK$
   a. $\angle U \cong$?
   b. $\overline{DB} \cong$?
   c. $NDBA \cong$?

4. In $\triangle MAP$ and $\triangle TIE$, $\angle A \cong \angle I$ and $\angle P \cong \angle E$.
   a. What is the relationship between $\angle M$ and $\angle T$?
   b. If $m\angle A = 52$ and $m\angle P = 36$, what is $m\angle T$?

Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. **Open-Ended** When do you think you might need to know that things are congruent in your everyday life?

6. If each angle in one triangle is congruent to its corresponding angle in another triangle, are the two triangles congruent? Explain.

7. **Error Analysis** Walter sketched the diagram below. He claims it shows that the two polygons are congruent. What information is missing to support his claim?
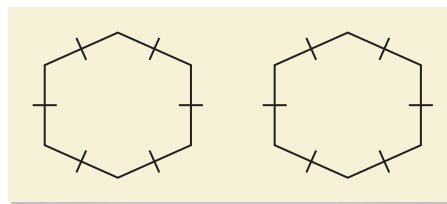
(A) Practice

8. Construction Builders use the king post truss (below left) for the top of a simple structure. In this truss, $\triangle ABC \cong \triangle ABD$. List the congruent corresponding parts.

See Problem 1.
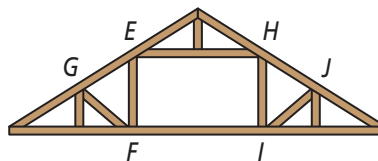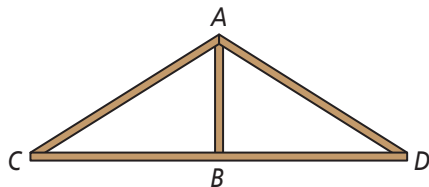
9. The attic frame truss (above right) provides open space in the center for storage. In this truss, $\triangle EFG \cong \triangle HIJ$. List the congruent corresponding parts.

$\triangle LMC \cong \triangle BJK$**. Complete the congruence statements.**

10. $\overline{LC} \cong$ __?__

11. $\overline{KJ} \cong$ __?__

12. $\overline{JB} \cong$ __?__

13. $\angle L \cong$ __?__

14. $\angle K \cong$ __?__

15. $\angle M \cong$ __?__

16. $\triangle CML \cong$ __?__

17. $\triangle KBJ \cong$ __?__

18. $\triangle MLC \cong$ __?__

19. $\triangle JKB \cong$ __?__
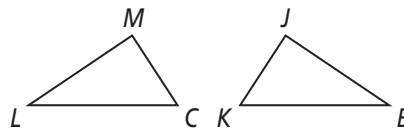
$POLY \cong SIDE$**. List each of the following.**

20. four pairs of congruent sides
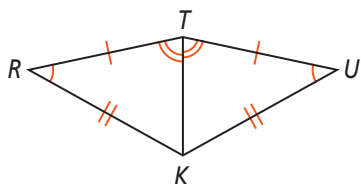
21. four pairs of congruent angles

At an archeological site, the remains of two ancient step pyramids are congruent. If $ABCD \cong EFGH$, find each of the following. (Diagrams are not to scale.)

See Problem 2.

22. AD

23. GH

24. $m\angle GHE$

25. $m\angle BAD$

26. EF

27. BC

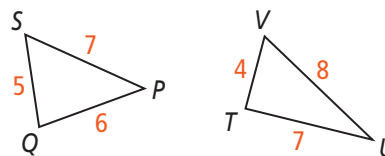**28.** $m\angle DCB$

29. $m\angle EFG$
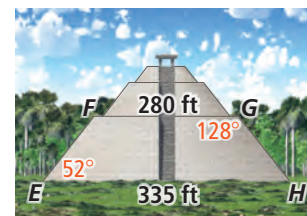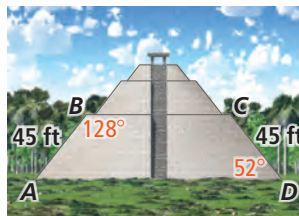
For Exercises 30 and 31, can you conclude that the triangles are congruent? Justify your answers.

See Problem 3.
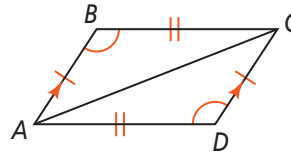
30. $\triangle TRK$ and $\triangle TUK$

31. $\triangle SPQ$ and $\triangle TUV$

32. Given: $\overline{AB} \parallel \overline{DC}$, $\angle B \cong \angle D$,
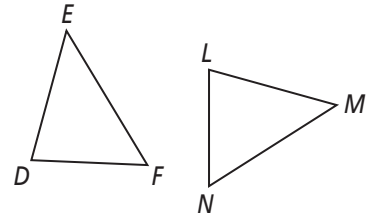$\overline{AB} \cong \overline{DC}$, $\overline{BC} \cong \overline{AD}$

Proof

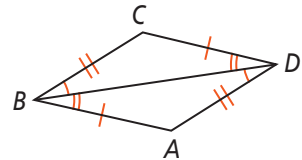**Prove:** $\triangle ABC \cong \triangle CDA$

See Problem 4.

B Apply

33. If $\triangle DEF \cong \triangle LMN$, which of the following must be a correct congruence statement?

Ⓐ $\overline{DE} \cong \overline{LN}$ Ⓒ $\angle N \cong \angle F$

Ⓑ $\overline{FE} \cong \overline{NL}$ Ⓓ $\angle M \cong \angle F$

34. Reasoning Randall says he can use the information in the figure to prove $\triangle BCD \cong \triangle DAB$. Is he correct? Explain.
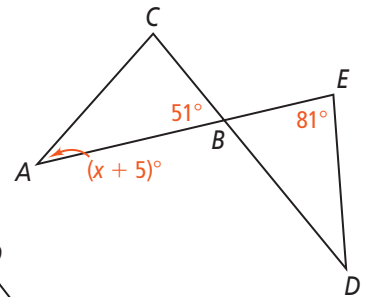
Algebra $\triangle ABC \cong \triangle DEF$. **Find the measures of the given angles or the lengths of the given sides.**
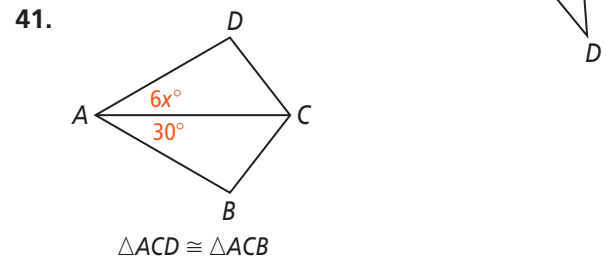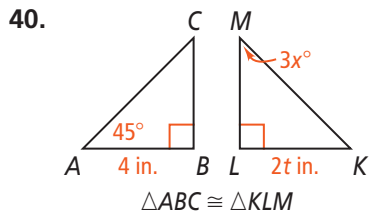
35. $m\angle A = x + 10$, $m\angle D = 2x$ **36.** $m\angle B = 3y$, $m\angle E = 6y - 12$

37. $BC = 3z + 2$, $EF = z + 6$ **38.** $AC = 7a + 5$, $DF = 5a + 9$

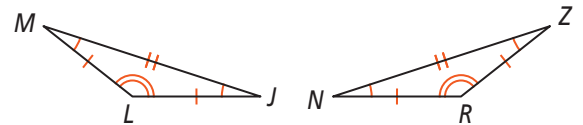**39. Think About a Plan** $\triangle ABC \cong \triangle DBE$. Find the value of x.

- What does it mean for two triangles to be congruent?
- Which angle measures do you already know?
- How can you find the missing angle measure in a triangle?

Algebra **Find the values of the variables.**

40.

$\triangle ABC \cong \triangle KLM$

41.

$\triangle ACD \cong \triangle ACB$

42. Complete in two different ways:
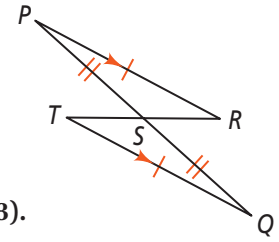$\triangle JLM \cong \underline{\ ?\ }$.

43. Open-Ended Write a congruence statement for two triangles. List the congruent sides and angles.

44. Given: $\overline{AB} \perp \overline{AD}$, $\overline{BC} \perp \overline{CD}$, $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$, $\overline{AB} \parallel \overline{CD}$

Proof **Prove:** $\triangle ABD \cong \triangle CDB$
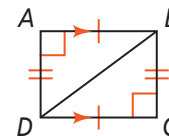
45. Given: $\overline{PR} \parallel \overline{TQ}$, $\overline{PR} \cong \overline{TQ}$, $\overline{PS} \cong \overline{QS}$, $\overline{PQ}$ bisects $\overline{RT}$

Proof **Prove:** $\triangle PRS \cong \triangle QTS$

Ⓒ **46. Writing** The 225 cards in Tracy's sports card collection are rectangles of three different sizes. How could Tracy quickly sort the cards?

Ⓒ **Challenge**   **Coordinate Geometry** The vertices of $\triangle GHJ$ are $G(-2, -1)$, $H(-2, 3)$, and $J(1, 3)$.

47. $\triangle KLM \cong \triangle GHJ$. Find KL, LM, and KM.

48. If L and M have coordinates $L(3, -3)$ and $M(6, -3)$, how many pairs of coordinates are possible for K? Find one such pair.
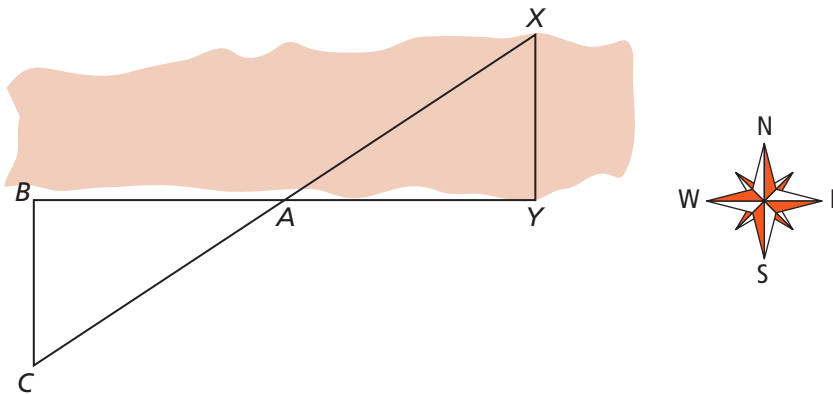
49. a. How many quadrilaterals (convex and concave) with different shapes or sizes can you make on a three-by-three geoboard? Sketch them. One is shown at the right.
   b. How many quadrilaterals of each type are there?
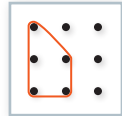
Apply What You've Learned

Look back at the information given on page 217 about how Jamal located the points in the diagram. The diagram is shown again below.

a. Copy and label the diagram. Include all the given information in your diagram.

b. Which angles do you know to be congruent? Explain.

c. Which sides do you know to be congruent? Explain.

d. Can you conclude that $\triangle ABC \cong \triangle AYX$ using the definition of congruent triangles? If not, what additional information would you need?

Building Congruent Triangles

Can you use shortcuts to find congruent triangles? Find out by building and comparing triangles.

Activity 1

Step 1 Cut straws into three pieces of lengths 4 in., 5 in., and 6 in. Thread a string through the three pieces of straw. The straw pieces can be in any order.

Step 2 Bring the two ends of the string together to make a triangle. Tie the ends to hold your triangle in place.

Step 3 Compare your triangle with your classmates' triangles. Try to make your triangle fit exactly on top of the other triangles.

1. Is your triangle congruent to your classmates' triangles?

2. Make a Conjecture What seems to be true about two triangles in which three sides of one are congruent to three sides of another?

3. As a class, choose three different lengths and repeat Steps 1–3. Are all the triangles congruent? Does this support your conjecture from Question 2?
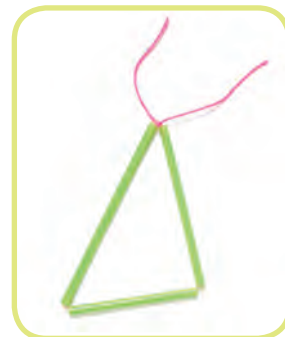
Activity 2

Step 1 Use a straightedge to draw and label any $\triangle ABC$ on tracing paper.

Step 2 Use a ruler. Carefully measure $\overline{AB}$ and $\overline{AC}$. Use a protractor to measure the angle between them, $\angle A$.

Step 3 Write the measurements on an index card and swap cards with a classmate. Draw a triangle using only your classmate's measurements.

Step 4 Compare your new triangle to your classmate's original $\triangle ABC$. Try to make your classmate's $\triangle ABC$ fit exactly on top of your new triangle.

4. Is your new triangle congruent to your classmate's original $\triangle ABC$?

5. Make a Conjecture What seems to be true about two triangles when they have two congruent sides and a congruent angle between them?

6. Make a Conjecture At least how many triangle measurements must you know in order to guarantee that all triangles built with those measurements will be congruent?